

The systematic study of the isotopic dependence of fusion dynamics for neutron and proton-rich nuclei using proximity formalism

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Abstract

The behaviors of barrier characteristics and fusion cross sections are analyzed by changing neutron over wide range of colliding systems. For this purpose, we have extended our previous study (Eur. Phys. J. A **48**, 21 (2012), it is devoted to the colliding systems with neutron-rich nuclei) to 125 isotopic systems with condition of $0.5 \leq N/Z \leq 1.6$ for their compound nuclei. The AW 95, Bass 80, Denisov DP and Prox. 2010 potentials are used to calculate the nuclear part of interacting potential. The obtained results show that the trend of barrier heights V_B and positions R_B as well as nuclear V_N and Coulomb V_C potentials (at $R = R_B$) as a function of $(N/Z - 1)$ quantity are non-linear (second-order) whereas the fusion cross sections follow a linear-dependence.

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1. INTRODUCTION

The systematic study of the isotopic dependence of interacting potential and fusion cross sections is one of the interesting subjects in nuclear physics. It is carried out using different theoretical models such as Skyrme energy density formalism, Ngô and Ngô, Christensen and Winther, Bass and Denisov potentials [1-3]. Although, the choice of these potentials was not made on merit to reproduce the experimental data. One can divide the isotopic systems in previous studies into three series: different collisions of Ca and Ni isotopes, with conditions of (i) $1 \leq N/Z \leq 2$ [1], (ii) $0.6 \leq N/Z \leq 2$ [2] and (iii) $0.5 \leq N/Z \leq 2$ [3]. In these conditions N and Z are neutron and proton numbers of compound nucleus. In all mentioned investigations, the isotopic dependence of barrier heights V_B and positions R_B as well as fusion cross sections σ_{fus} versus N/Z ratio, has been examined. The obtained results show that the increasing trend of R_B and σ_{fus} and decreasing trend of V_B as a function of N/Z ratio for fusion systems with condition (i) are linear, see fig. 2 of Ref. [1]. On the other hand, in different collisions of Ca and Ni isotopes which have condition of $0.6 \leq N/Z \leq 2$ (or $0.5 \leq N/Z \leq 2$), it is shown that this behavior is non-linear (second-order) for R_B and V_B whereas the fusion cross sections follow a linear-dependence (see corresponding figures of Refs. [2,3]). As a result, one expects that with increase of neutron and its effect on the attractive force, the Coulomb barrier heights decrease. Since the fusion probability is directly dependent on these parameters, therefore it is predictable that the fusion cross sections increase with addition of neutron in different isotopic systems. As an important issue, it can be noted that the experimental data have only been reported for fusion reactions with nuclei which are near the stability line ($N = Z$). Therefore, one can compare the measured and calculated values of fusion cross sections for limited numbers of reactions. However, using the proposed semi-empirical approaches, such as Ref. [4], the experimental data are well described.

In a recent study, we have analyzed the isotopic dependence of fusion cross sections and barrier characteristics for 50 fusion reactions with condition of $1 \leq N/Z \leq 1.6$ [5]. For this purpose, we have selected the fusion reactions which the C, O, Mg, Si, S, Ca, Ar, Ti and

Ni isotopes are as their participant nuclei. In this study, the nuclear part of interacting potential has been calculated using four versions of proximity model, namely AW 95 [6], Bass 80 [7,8], Denisov DP [9] and Prox. 2010 [10] potentials. It is shown that these models have good agreement with experimental data [10-12]. Our obtained results, like Ref. [1], confirmed the linear trend of R_B , V_B and σ_{fus} versus N/Z ratio for fusion reactions with $1 \leq N/Z \leq 1.6$.

In present work, we are going to extend our studies to both proton and neutron-rich systems. For this purpose, we have chosen 125 fusion reactions so that all colliding pairs are assumed to be spherical and the N/Z ratio of compound nuclei, which are formed during fusion process, to be in the range $0.5 \leq N/Z \leq 1.6$. For neutron-deficient systems, we have only intended to investigate the influence of changing neutron on the input potential channel and fusion probabilities. The lightest (^{10}C) and heaviest (^{54}Ni) proton-rich nuclei have the half-life $T_{1/2} = 19.30$ s and $T_{1/2} = 104.00$ ms, respectively. The proximity formalism and Wong model [13] are employed to calculate the nuclear potentials and fusion cross sections, respectively.

In summery, our motivations in this work are: i) There is no systematic study on the isotopic dependence of fusion cross sections based on the AW 95, Bass 80, Denisov and Prox. 2010 potentials in the range of $N/Z \leq 1$. The applied models in previous studies [1-3] were not made on merit to reproduce the experimental data whereas our selected models are able to reproduce experimental data within %10, on the average, see Refs. [10-13] for details. ii) The different colliding systems in previous works [1-3] are only included Ca and Ni isotopes. Whereas, we have used the C, O, Mg, Si, S, Ca, Ar, Ti and Ni nuclei as colliding pairs which have been taken from proton-rich region of periodic table. Our selections can be more appropriate for better understanding of the isotopic dependence of R_B , V_B and σ_{fus} versus N/Z ratio. iii) The isotopic systems which are used in previous studies such as Refs [1-3] lying far from the stability line ($N=Z$) and in these regions the experimental data have not been reported. Whereas, using present study, one can analyze the calculated results and compare them with corresponding experimental data. iv) Such studies can be very useful to predict the properties of new and superheavy elements which are produced in fusion process

and are not available at present.

The study of neutron rich nuclei is also reported at heavy-ion collisions with intermediate energies. The effects of isospin degree of freedom in collective and elliptic flow have been studied, for example, in Refs. [14-17]. Recently, using the dynamic approach based on the macroscopic models, the isospin effects have been examined for $^{40}\text{Ca}+^{90,96}\text{Zr}$, $^{48}\text{Ca}+^{90}\text{Zr}$ fusion reactions [18]. The obtained results reveal that the dynamic effects decrease barrier height and thickness.

The paper is organized as follows: in sections 2, we discuss about the nuclear part of the total interaction potential as well as the employed models for calculation it. The analysis of isotopic dependence of barrier characteristics and fusion cross sections in different ranges of N/Z ratio have been carried out in Secs. 3 and 4. Section 5 is devoted to some concluding remarks.

2. THE NUCLEAR POTENTIAL OF INTERACTING SYSTEMS

In general, with assumption that the participant nuclei in fusion reaction to be in the s-wave state ($\ell = 0$), the total potential can be defined as the sum of two parts which are caused by electrostatic (Coulomb-repulsion) and strong (nuclear-attraction) interactions. In recent years, many theoretical models have been introduced to parameterize the last interactions. The proximity formalism is one of the useful models for calculating of nuclear potential. The various versions of this formalism are introduced in [10-12]. These studies have been carried out on the many different systems. The obtained results show that all introduced models determine the fusion barrier heights with accuracy $\pm 10\%$, on the average. Among various versions, our selected potentials, namely AW 95, Bass 80, Denisov DP and Prox. 2010, reproduce the best results for potential and fusion cross sections. These models are briefly explained in the following.

Aage winther proposed a nuclear potential which is parameterized based on the Woods-Saxon form [6],

$$V_N^{AW95}(r) = -\frac{V_0}{1 + \exp(\frac{r-R_0}{a})}, \quad (1)$$

here, the V_0 , R_0 and a parameters are defined as,

$$V_0 = 16\pi \frac{R_1 R_2}{R_1 + R_2} \gamma a, \quad (2)$$

$$R_0 = R_1 + R_2, \quad (3)$$

and

$$a = \left[\frac{1}{1.17(1 + 0.53(A_1^{-1/3} + A_2^{-1/3}))} \right]. \quad (4)$$

The R_i and γ parameters in equations (2) and (3) respectively stand for the radius of target/projectile and surface energy coefficient and can be written as,

$$R_i = 1.2A_i^{1/3} - 0.09 \quad (i = 1, 2), \quad (5)$$

$$\gamma = 0.95 \left[1 - 1.8 \left(\frac{N_p - Z_p}{A_p} \right) \left(\frac{N_t - Z_t}{A_t} \right) \right]. \quad (6)$$

where $A_{p(t)}$, $Z_{p(t)}$ and $N_{p(t)}$ are characteristics of target and projectile.

According to the proximity theorem [19], the nuclear potentials which are based on the models such as Bass 80, Denisov and Porx. 2010 define as the product of a geometrical factor and a universal function which are respectively dependent on the mean curvature of the interaction surface and the separation distance. Therefore, one can use the below equations to calculate the nuclear potential based on the selected models,

$$V_N^{Bass80}(r) = -\frac{R_1 R_2}{R_1 + R_2} \Phi(s = r - R_1 - R_2), \quad (7)$$

$$\begin{aligned} V_N^{DenisovDP}(r) = & -1.989843 \frac{R_1 R_2}{R_1 + R_2} \Phi(s = r - R_1 - R_2 - 2.65) \\ & \times \left[1 + 0.003525139 \left(\frac{A_1}{A_2} + \frac{A_2}{A_1} \right)^{3/2} - 0.4113263(I_1 + I_2) \right], \end{aligned} \quad (8)$$

$$V_N^{Prox.2010}(r) = 4\pi\gamma \frac{R_1 R_2}{R_1 + R_2} \Phi(s = r - C_1 - C_2). \quad (9)$$

In these relations the radius parameter R_i can be written as follows,

$$R_i^{Bass80} = R_s \left(1 - \frac{0.98}{R_s^2} \right) \quad (i = 1, 2), \quad (10)$$

$$R_i^{DenisovDP} = 1.2332 A_i^{1/3} (1 + 2.348443/A_i - 0.151541 A_{si}) \quad (i = 1, 2), \quad (11)$$

where the sharp radius R_s in Eq. (10) is given as $R_s = 1.28 A^{1/3} - 0.76 + 0.8 A^{-1/3}$. The R_i parameter in Prox. 2010 is similar to AW 95 model, Eq. (5). The universal function $\Phi(s)$ is respectively defined by Eqs. (20), (42) and (6) of Ref. [11] for Bass 80, Denisov DP and Prox. 2010 potentials.

3. ISOTOPIC ANALYSIS OF TOTAL POTENTIALS

By adding the Coulomb part to our selected nuclear potentials, which are introduced in previous sections, one can calculate total interaction potential for different fusion systems by following simple expression,

$$V_{tot}(r) = V_N(r) + V_C(r) = V_N(r) + \frac{Z_1 Z_2 e^2}{r}, \quad (12)$$

where Z_1 and Z_2 are atomic numbers of interaction nuclei. The effects of addition/removal of neutron on the various potentials are shown in Fig. 1. In this figure, the nuclear $V_N(r)$, Coulomb $V_C(r)$ and total $V_T(r)$ potentials are separately plotted for $A_1\text{Ni} + A_2\text{Ni}$ isotopic systems, which are consist of proton-rich ($^{48}\text{Ni} + ^{48}\text{Ni}$, $^{50}\text{Ni} + ^{50}\text{Ni}$ and $^{54}\text{Ni} + ^{54}\text{Ni}$), neutron-rich ($^{58}\text{Ni} + ^{58}\text{Ni}$ and $^{64}\text{Ni} + ^{64}\text{Ni}$) as well as symmetric ($^{56}\text{Ni} + ^{56}\text{Ni}$) participant nuclei. Using the results shown in figure 1, the influence of neutron-excess on the shape of considered potentials is quite evident.

The exact values of the barrier height V_B^{theor} and barrier position R_B^{theor} have been extracted using

$$\left(\frac{dV_{tot}(r)}{dr}\right)_{r=R_B^{theor}} = 0 \quad ; \quad \left(\frac{d^2V_{tot}(r)}{dr^2}\right)_{r=R_B^{theor}} \leq 0. \quad (13)$$

In the beginning, we have calculated the barrier heights and positions for different fusion systems using four-type of proximity potentials, namely AW 95, Bass 80, Denisov DP and Prox. 2010. The obtained results of these calculations are listed in Table 1. It can be seen that with addition of neutron in the interacting systems, the barrier heights and positions, respectively, decrease and increase. Moreover, using definition of Coulomb potential $V_C(r) = Z_1 Z_2 e^2 / r$, we expect that by increasing R_B in any isotopic system the values of $V_C(r = R_B)$ reduce.

To get a better comparison between experimental and theoretical values of barrier heights and positions, we have displayed the V_B^{theor} vs V_B^{exp} and the R_B^{theor} vs R_B^{exp} , see Figs. 2 and 3. It is shown that the calculated values of V_B^{theor} based on the considered models have good compatibility with experimental data. On the other hand, according to Fig. 3, one can't find a regular behavior in predictions of barrier positions. This may be caused by the large uncertainties of these values.

3.1 The isotopic dependence of barrier heights V_B and positions R_B

The percentage difference of barrier characteristics, i.e. $\Delta R_B(\%)$ and $\Delta V_B(\%)$, are defined as follows to study the isotopic dependence of barrier heights and positions,

$$\Delta R_B(\%) = \frac{R_B - R_B^0}{R_B^0} \times 100, \quad (14)$$

$$\Delta V_B(\%) = \frac{V_B - V_B^0}{V_B^0} \times 100. \quad (15)$$

Above, R_B^0 and V_B^0 are barrier characteristics of $N = Z$ case. Indeed, using the proposed producer of Refs. [1-3], our criterion to analyze the trend of R_B and V_B versus N/Z ratio in each set of colliding systems is the symmetric reaction of that set. Using the straight-line interpolation between know values, we have estimated the barrier characteristics (R_B and V_B)

for symmetric reactions ($N = Z$) that these values aren't available for them. The obtained results for Eqs. (14) and (15) based on the selected proximity potentials have been plotted in Fig. 4. It is clear that with addition of neutron, the values of R_B and V_B respectively increase and decrease. Moreover, figure 4 shows the regular behaviors for barrier heights and positions. One can analyze these behaviors using the following ranges of N/Z ratio.

3.1.1 The ranges of $0.5 \leq N/Z \leq 1$ and $1 \leq N/Z \leq 1.6$

The values of barrier heights and positions based on the considered proximity potentials follow a linear-dependence for either proton ($0.5 \leq N/Z \leq 1$) or neutron-rich ($1 \leq N/Z \leq 1.6$) systems (see Fig. 4). One can parameterize the percentage difference of R_B and V_B , which are calculated by Eqs. (14) and (15), using the below forms in two mentioned regions, namely for $N/Z \leq 1$,

$$\Delta R_B(\%) = \alpha_1 \left(\frac{N}{Z} - 1 \right); \quad \Delta V_B(\%) = \alpha_2 \left(\frac{N}{Z} - 1 \right), \quad (16)$$

and for $N/Z \geq 1$,

$$\Delta R_B(\%) = \alpha'_1 \left(\frac{N}{Z} - 1 \right), \quad \Delta V_B(\%) = \alpha'_2 \left(\frac{N}{Z} - 1 \right), \quad (17)$$

where the values of the constants α_i and α'_i , for $i=1, 2$, have been listed in Table 2.

3.1.2 The range of $0.5 \leq N/Z \leq 1.6$

In whole region of $0.5 \leq N/Z \leq 1.6$, the behavior of heights and positions of the barrier are non-linear and the percentage difference of these values can be parameterized by a second-order form (see figure 4),

$$\Delta R_B(\%) = \beta_1 \left(\frac{N}{Z} - 1 \right) + \beta_2 \left(\frac{N}{Z} - 1 \right)^2; \quad \Delta V_B(\%) = \beta_3 \left(\frac{N}{Z} - 1 \right) + \beta_4 \left(\frac{N}{Z} - 1 \right)^2 \quad (18)$$

where the values of the coefficients β_i have been listed in Table 3. In Fig. 4, we have also plotted the results of two theoretical models [3,20], for example. It is clear that the calculated values of $\Delta R_B(\%)$ and $\Delta V_B(\%)$ for these models are consistent with our predictions. On the other hand, the results shown in Fig. 4 confirm the trend of R_B and V_B which are reported in Refs. [2,3].

3.2 The isotopic dependence of nuclear V_N and Coulomb V_C potentials

In addition to R_B and V_B , we have interested to analyze the isotopic dependence of nuclear and Coulomb potentials (at $r = R_B$) by changing neutron. For this aim, one should calculate the values of $\Delta V_N(\%)$ and $\Delta V_C(\%)$ using the following relations,

$$\Delta V_N(\%) = \frac{V_N - V_N^0}{V_N^0} \times 100. \quad (19)$$

$$\Delta V_C(\%) = \frac{V_C - V_C^0}{V_C^0} \times 100. \quad (20)$$

where V_N^0 and V_C^0 are the values of nuclear and Coulomb potentials for symmetric reaction, see Fig. 5. Similar previous calculations of R_B and V_B and using the above suggested manner, one can predict the values of $V_N(r = R_B)$ and $V_C(r = R_B)$ for symmetric reactions that these values aren't available for them. Because of increasing neutron, it is predictable that the values of nuclear and Coulomb potentials for different isotopic systems increase and decrease, respectively. The regular behaviors of V_N and V_C at $r = R_B$ are examined in following ranges.

3.2.1 The ranges of $0.5 \leq N/Z \leq 1$ and $1 \leq N/Z \leq 1.6$

The values of Coulomb and nuclear potentials based on the AW 95, Bass 80, Denisov DP and Prox. 2010 potentials follow a linear-dependence for either proton ($0.5 \leq N/Z \leq 1$) or neutron-rich ($1 \leq N/Z \leq 1.6$) systems (see Fig. 5). The percentage differences of V_C and

V_N are parameterized as following forms for $N/Z \leq 1$,

$$\Delta V_C(\%) = \alpha_3 \left(\frac{N}{Z} - 1 \right), \quad \Delta V_N(\%) = \alpha_4 \left(\frac{N}{Z} - 1 \right), \quad (21)$$

and for $N/Z \geq 1$,

$$\Delta V_C(\%) = \alpha'_3 \left(\frac{N}{Z} - 1 \right); \quad \Delta V_N(\%) = \alpha'_4 \left(\frac{N}{Z} - 1 \right), \quad (22)$$

where the values of constants α_i and α'_i , for $i=3, 4$, have been listed in Table 2.

3.2.2 The range of $0.5 \leq N/Z \leq 1.6$

In range of $0.5 \leq N/Z \leq 1.6$, we have found a non-linear regular behavior for both nuclear and Coulomb potentials. One can formulate these trends as,

$$\Delta V_C(\%) = \beta'_1 \left(\frac{N}{Z} - 1 \right) + \beta'_2 \left(\frac{N}{Z} - 1 \right)^2; \quad \Delta V_N(\%) = \beta'_3 \left(\frac{N}{Z} - 1 \right) + \beta'_4 \left(\frac{N}{Z} - 1 \right)^2, \quad (23)$$

where the values of coefficients β'_i have been listed in Table 3.

4. ISOTOPIC ANALYSIS OF FUSION CROSS SECTIONS

In present study, one dimensional penetration model [21,22] is used to study the fusion cross section, σ_{fus} . In this formalism, the σ_{fus} defines as the summation on the quantum-mechanical transmission probability through the potential barrier for a specified angular momentum l and center-of-mass energy, namely $T_l(E_{c.m.})$,

$$\sigma_{fus} = \frac{\pi \hbar^2}{2\mu E_{c.m.}} \sum_{l=0}^{l_{max}} (2l+1) T_l(E_{c.m.}), \quad (24)$$

where in this relation μ is the reduced mass of interacting system. With assumption that $E_{c.m.} \gg V_B$, the Eq. (24) reduces to the well-known sharp cutoff formula,

$$\sigma_{fus} = 10\pi R_B^2 \left(1 - \frac{V_B}{E_{c.m.}} \right). \quad (25)$$

Among introduced potentials in Refs. [10-12], we have selected the models that have the best agreement with fusion data. To get a better comparison, the fusion cross sections for $^{41}\text{Ca}+^{42}\text{Ti}$ system, namely $^{40}\text{Ca}+^{46}\text{Ti}$, $^{40}\text{Ca}+^{48}\text{Ti}$ and $^{40}\text{Ca}+^{50}\text{Ti}$ fusion reactions, are calculated by sharp cutoff formula, Eq. (25). For this purpose, we have used the obtained results of R_B and V_B based on the AW 95, Bass 80, Denisov DP and Prox. 2010 potentials (see Fig. 6). In this figure, the corresponding experimental data for considered reactions are taken from [23]. It is shown that the predicted values for fusion cross sections are consistent with experimental data particularly at above barrier energies.

To systematic study of fusion cross section in isotopic systems, we have defined the percentage difference of this quantity, namely $\Delta\sigma_{fus}(\%)$, as following form,

$$\Delta\sigma_{fus}(\%) = \frac{\sigma_{fus}(E_{c.m.}^0) - \sigma_{fus}^0(E_{c.m.}^0)}{\sigma_{fus}^0(E_{c.m.}^0)} \times 100, \quad (26)$$

where the $\sigma_{fus}(E_{c.m.}^0)$ is fusion cross section for reaction with $N = Z$ condition. We have computed the $\Delta\sigma_{fus}(\%)$ at some above barrier energies such as $E_{c.m.} = 1.125V_B^0$ and $E_{c.m.} = 1.375V_B^0$. The calculated results have been shown in Fig. (7). It is clear that by increasing neutron in interacting systems and decreasing barrier height, one expects that fusion cross sections enhance. This behavior is quite obvious in Fig. (7). In contrast the obtained results for barrier characteristics, the relationship between variations of fusion cross sections $\Delta\sigma_{fus}(\%)$ and increasing of $(N/Z - 1)$ quantity is linear. This subject is accurate for all neutron and proton-rich systems ($0.5 \leq N/Z \leq 1.6$). We have parameterized this linear trend of fusion cross sections as following form,

$$\Delta\sigma_{fus}(\%) = \gamma \left(\frac{N}{Z} - 1 \right), \quad (27)$$

where the values of constant coefficient γ have been listed in Table 4 for various potentials and energies.

5. CONCLUSIONS

In this paper, using the systematic study on the large range of colliding pairs C, O, Mg, Si, S, S, Ca, Ar, Ti and Ni with $84 \leq Z_1 Z_2 \leq 784$, we have analyzed the isotopic dependence of different parameters of interacting potentials and fusion cross sections. For calculating of these values, we have respectively used four confirmed versions of proximity formalism and Wong model. Our obtained results for three considered regions of N/Z ratio, i.e. $0.5 \leq N/Z \leq 1$, $1 \leq N/Z \leq 1.6$ and $0.5 \leq N/Z \leq 1.6$ are: a) For fusion systems with condition of $0.5 \leq N/Z \leq 1$, the variations trend of barrier characteristics, R_B , $V_C(R = R_B)$, $V_N(R = R_B)$ and V_B , with respect to corresponding symmetric reaction ($N = Z$), follow a linear dependence as a function of $(N/Z-1)$. b) For colliding systems with condition of $1 \leq N/Z \leq 1.6$, the above quantities can be parameterized as linear. c) In whole range $0.5 \leq N/Z \leq 1.6$, the values of ΔR_B , ΔV_C , ΔV_N , and ΔV_B have different trends. In other words, these values follow a non-linear second order behavior with addition/removal neutron.

As a common property, one can point out the values of R_B , $V_N(R = R_B)$ as well as V_B , $V_C(R = R_B)$, respectively, increase and decrease with increasing of neutron. On the other hand, the fusion cross sections follow a linear dependence for all considered isotopic systems.

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FIGURE CAPTIONS

Fig. 1. The various components of interacting potential, i.e. nuclear V_N , Coulomb V_C and total V_T potentials, versus inter-nuclear distance r (in fm) for $^{A_1}\text{Ni} + ^{A_2}\text{Ni}$ isotopic system. These calculations are based on the Prox. 2010 and Denisov DP potentials, for example.

Fig. 2. The obtained theoretical barrier heights V_B^{theor} (in MeV) as a function of corresponding experimental data V_B^{exp} (in MeV) [24-40] based on the AW 95, Bass 80, Denisov DP and Prox. 2010 potentials.

Fig. 3. The obtained theoretical barrier positions R_B^{theor} (in MeV) as a function of corresponding experimental data R_B^{exp} (in MeV) [24-40] based on the AW 95, Bass 80, Denisov DP and Prox. 2010 potentials.

Fig. 4. The variations trend of R_B ($\Delta R_B(\%)$, left panels) and V_B ($\Delta V_B(\%)$, right panels) as a function of $(N/Z - 1)$ based on the selected potentials AW 95, Bass 80, Denisov DP and Prox. 2010. The dash and short-dotted lines are respectively used to extract the linear-dependence of $\Delta R_B(\%)$ and $\Delta V_B(\%)$ values in $0.5 \leq N/Z \leq 1$ and $1 \leq N/Z \leq 1.6$ regions. The solid lines are caused by the non-linear (second order) fitting to the calculated values in whole range of $0.5 \leq N/Z \leq 1.6$.

Fig. 5. The variations trend of V_N ($\Delta V_N(\%)$, left panels) and V_C ($\Delta V_C(\%)$, right panels) as a function of $(N/Z - 1)$ based on the selected potentials AW 95, Bass 80, Denisov DP and Prox. 2010. The dash and short-dotted lines are respectively used to extract the linear-dependence of $\Delta R_B(\%)$ and $\Delta V_B(\%)$ values in $0.5 \leq N/Z \leq 1$ and $1 \leq N/Z \leq 1.6$ regions. The solid lines are caused by the non-linear (second order) fitting to the calculated values in whole range of $0.5 \leq N/Z \leq 1.6$.

Fig. 6 The comparison of theoretical, Eq. (25), and experimental data [23] for fusion

cross sections based on the various versions of proximity formalism, namely AW 95, Bass 80, Denisov DP and Prox. 2010 potentials. These calculations have been carried out for $^{40}\text{Ca}+^{46}\text{Ti}$, $^{40}\text{Ca}+^{48}\text{Ti}$ and $^{40}\text{Ca}+^{50}\text{Ti}$ fusion reactions.

Fig. 7. The percentage difference of fusion cross sections $\Delta\sigma_{fus}(\%)$ as a function of $(N/Z-1)$ for different colliding systems. These values are calculated for two instance of above barrier energies, namely $E_{c.m.} = 1.125V_B^0$ (left panels) and $E_{c.m.} = 1.375V_B^0$ (right panels), which are based on the (a) AW 95, (b) Bass 80, (c) Denisov DP, and (d) Prox. 2010 potentials. The linear-behavior of σ_{fus} have been parameterized by solid-lines (Eq. (27)).

TABLE CAPTIONS

Table 1. The obtained values for barrier positions R_B and heights V_B based on the AW 95, Bass 80, Denisov DP and Prox. 2010 potentials for neutron-deficient and -rich systems.

Reaction	N/Z	R_B^a (fm)	V_B^a (MeV)	R_B^b (fm)	V_B^b (MeV)	R_B^c (fm)	V_B^c (MeV)	R_B^d (fm)	V_B^d (MeV)
$^{10}\text{C}+^{22}\text{Si}$	0.6	7.76	14.35	7.78	11.14	7.40	14.76	7.34	14.79
$^{12}\text{C}+^{22}\text{Si}$	0.7	8.08	13.82	8.04	13.73	7.81	14.06	7.75	14.09
$^{12}\text{C}+^{24}\text{Si}$	0.8	8.22	13.61	8.19	13.51	8.02	13.71	7.98	13.73
$^{12}\text{C}+^{26}\text{Si}$	0.9	8.35	13.41	8.32	13.32	8.20	13.41	8.15	13.47
$^{12}\text{C}+^{28}\text{Si}$	1	8.47	13.23	8.45	13.14	8.36	13.15	8.29	13.26
$^{12}\text{C}+^{29}\text{Si}$	1.05	8.53	13.15	8.51	13.05	8.43	13.04	8.35	13.18
$^{12}\text{C}+^{30}\text{Si}$	1.1	8.58	13.07	8.57	12.97	8.50	12.92	8.41	13.10
$^{12}\text{O}+^{20}\text{Mg}$	0.6	7.73	16.43	7.76	16.18	7.24	17.09	7.33	16.94
$^{12}\text{O}+^{22}\text{Mg}$	0.7	7.94	16.05	7.92	15.89	7.50	16.62	7.63	16.33
$^{12}\text{O}+^{24}\text{Mg}$	0.8	8.12	15.72	8.07	15.64	7.70	16.23	7.85	15.92
$^{14}\text{O}+^{24}\text{Mg}$	0.9	8.33	15.34	8.30	15.26	8.08	15.53	8.12	15.45
$^{16}\text{O}+^{24}\text{Mg}$	1	8.52	15.02	8.50	14.93	8.37	15.01	8.34	15.08
$^{16}\text{O}+^{26}\text{Mg}$	1.1	8.65	14.81	8.65	14.72	8.53	14.72	8.47	14.88
$^{18}\text{O}+^{24}\text{Mg}$	1.1	8.70	14.74	8.68	14.65	8.60	14.60	8.52	14.80
$^{12}\text{O}+^{22}\text{Si}$	0.545	7.72	19.18	7.77	18.83	7.18	20.04	7.21	20.03
$^{14}\text{O}+^{22}\text{Si}$	0.636	8.02	18.53	8.00	18.35	7.59	19.12	7.64	19.04
$^{16}\text{O}+^{22}\text{Si}$	0.727	8.27	18.02	8.21	17.94	7.92	18.45	7.95	18.36
$^{16}\text{O}+^{24}\text{Si}$	0.818	8.41	17.74	8.36	17.67	8.13	18.00	8.16	17.94
$^{16}\text{O}+^{26}\text{Si}$	0.909	8.54	17.49	8.50	17.41	8.32	17.62	8.33	17.62
$^{16}\text{O}+^{28}\text{Si}$	1	8.66	17.26	8.63	17.18	8.48	17.29	8.46	17.36
$^{16}\text{O}+^{29}\text{Si}$	1.045	8.72	17.15	8.69	17.07	8.56	17.14	8.52	17.25
$^{16}\text{O}+^{30}\text{Si}$	1.090	8.77	17.05	8.75	16.96	8.63	17.00	8.58	17.15
$^{18}\text{O}+^{28}\text{Si}$	1.090	8.83	16.94	8.81	16.86	8.72	16.82	8.64	17.03
$^{20}\text{Mg}+^{30}\text{S}$	0.786	8.66	29.50	8.60	29.42	8.25	30.35	8.40	29.97
$^{22}\text{Mg}+^{30}\text{S}$	0.857	8.82	29.01	8.76	28.95	8.48	29.59	8.59	29.37
$^{20}\text{Mg}+^{32}\text{S}$	0.857	8.79	29.10	8.72	29.07	8.40	29.85	8.55	29.50
$^{22}\text{Mg}+^{32}\text{S}$	0.928	8.94	28.65	8.88	28.60	8.63	29.12	8.73	28.96
$^{24}\text{Mg}+^{32}\text{S}$	1	9.08	28.24	9.03	28.18	8.84	28.51	8.88	28.50
$^{24}\text{Mg}+^{34}\text{S}$	1.071	9.18	27.94	9.14	27.88	8.98	28.10	8.99	28.20
$^{26}\text{Mg}+^{32}\text{S}$	1.071	9.21	27.87	9.17	27.80	9.02	27.99	9.01	28.12
$^{26}\text{Mg}+^{34}\text{S}$	1.143	9.31	27.60	9.28	27.50	9.16	27.60	9.11	27.84

(a) Based on the AW 95 potential (b) Based on the Bass 80 potential

(c) Based on the Denisov DP potential (d) Based on the Prox. 2010 potential

Table 1. (Continued)

Reaction	N/Z	R_B^a (fm)	V_B^a (MeV)	R_B^b (fm)	V_B^b (MeV)	R_B^c (fm)	V_B^c (MeV)	R_B^d (fm)	V_B^d (MeV)
$^{22}\text{Si}+^{22}\text{Si}$	0.571	8.18	31.70	8.19	31.35	7.68	33.09	7.73	32.98
$^{22}\text{Si}+^{24}\text{Si}$	0.643	8.37	31.06	8.34	30.85	7.90	32.26	8.01	31.94
$^{24}\text{Si}+^{24}\text{Si}$	0.714	8.54	30.50	8.49	30.37	8.12	31.47	8.24	31.12
$^{24}\text{Si}+^{26}\text{Si}$	0.786	8.69	30.01	8.63	29.93	8.31	30.80	8.43	30.48
$^{26}\text{Si}+^{26}\text{Si}$	0.857	8.83	29.57	8.77	29.51	8.49	30.16	8.60	29.94
$^{26}\text{Si}+^{28}\text{Si}$	0.928	8.97	29.17	8.91	29.12	8.66	29.61	8.75	29.48
$^{28}\text{Si}+^{28}\text{Si}$	1	9.09	28.80	9.04	28.74	8.85	29.08	8.89	29.07
$^{28}\text{Si}+^{30}\text{Si}$	1.071	9.20	28.46	9.16	28.40	9.00	28.61	9.01	28.72
$^{30}\text{Si}+^{30}\text{Si}$	1.143	9.31	28.15	9.29	28.06	9.16	28.17	9.12	28.41
$^{26}\text{Si}+^{52}\text{Ni}$	0.857	9.49	55.12	9.42	55.07	9.18	56.28	9.30	55.81
$^{26}\text{Si}+^{54}\text{Ni}$	0.904	9.57	54.70	9.50	54.66	9.28	55.77	9.39	55.32
$^{26}\text{Si}+^{56}\text{Ni}$	0.952	9.65	54.29	9.58	54.26	9.36	55.29	9.47	54.87
$^{28}\text{Si}+^{52}\text{Ni}$	0.904	9.62	54.41	9.55	54.39	9.36	55.32	9.44	55.04
$^{28}\text{Si}+^{54}\text{Ni}$	0.952	9.70	54.03	9.64	53.98	9.45	54.83	9.53	54.60
$^{28}\text{Si}+^{58}\text{Ni}$	1.048	9.84	53.30	9.79	53.22	9.62	53.94	9.68	53.81
$^{28}\text{Si}+^{62}\text{Ni}$	1.143	9.98	52.63	9.94	52.52	9.78	53.14	9.80	53.16
$^{28}\text{Si}+^{64}\text{Ni}$	1.190	10.04	52.31	10.01	52.19	9.85	52.78	9.86	52.88
$^{30}\text{Si}+^{58}\text{Ni}$	1.095	9.96	52.73	9.92	52.62	9.78	53.14	9.79	53.22
$^{30}\text{Si}+^{62}\text{Ni}$	1.190	10.09	52.10	10.07	51.94	9.94	52.37	9.92	52.61
$^{30}\text{Si}+^{64}\text{Ni}$	1.238	10.15	51.80	10.14	51.61	10.01	52.01	9.97	52.35
$^{34}\text{Ca}+^{34}\text{Ca}$	0.7	9.09	58.45	9.02	57.29	8.69	60.26	8.83	59.69
$^{36}\text{Ca}+^{36}\text{Ca}$	0.8	9.33	57.10	9.25	57.05	8.99	58.52	9.12	57.96
$^{38}\text{Ca}+^{38}\text{Ca}$	0.9	9.54	55.94	9.47	55.92	9.26	57.01	9.36	56.60
$^{40}\text{Ca}+^{34}\text{Ca}$	0.85	9.44	56.50	9.35	56.53	9.10	57.87	9.24	57.30
$^{40}\text{Ca}+^{36}\text{Ca}$	0.9	9.54	55.94	9.47	55.94	9.25	57.06	9.36	56.62
$^{40}\text{Ca}+^{38}\text{Ca}$	0.95	9.64	55.42	9.57	55.39	9.38	56.33	9.47	56.02
$^{40}\text{Ca}+^{40}\text{Ca}$	1	9.74	54.92	9.68	54.88	9.50	55.67	9.57	55.48
$^{40}\text{Ca}+^{44}\text{Ca}$	1.1	9.91	54.01	9.87	53.93	9.72	54.51	9.75	54.55
$^{40}\text{Ca}+^{48}\text{Ca}$	1.2	10.08	53.18	10.05	53.08	9.92	53.51	9.90	53.78
$^{48}\text{Ca}+^{48}\text{Ca}$	1.4	10.38	51.75	10.42	51.40	10.33	51.52	10.18	52.40

(a) Based on the AW 95 potential (b) Based on the Bass 80 potential

(c) Based on the Denisov DP potential (d) Based on the Prox. 2010 potential

Table 1. (Continued)

Reaction	N/Z	R_B^a (fm)	V_B^a (MeV)	R_B^b (fm)	V_B^b (MeV)	R_B^c (fm)	V_B^c (MeV)	R_B^d (fm)	V_B^d (MeV)
$^{38}\text{Ca}+^{38}\text{Ti}$	0.809	9.44	62.06	9.36	62.04	9.10	63.58	9.24	62.97
$^{38}\text{Ca}+^{40}\text{Ti}$	0.857	9.55	61.46	9.47	61.44	9.24	62.78	9.55	61.46
$^{38}\text{Ca}+^{42}\text{Ti}$	0.857	9.64	60.91	9.57	60.88	9.36	62.05	9.64	60.91
$^{38}\text{Ca}+^{44}\text{Ti}$	0.952	9.73	60.38	9.67	60.36	9.47	61.38	9.57	61.02
$^{40}\text{Ca}+^{38}\text{Ti}$	0.857	9.55	61.42	9.47	61.44	9.23	62.81	9.36	62.25
$^{40}\text{Ca}+^{40}\text{Ti}$	0.904	9.65	60.86	9.57	60.86	9.36	62.03	9.47	61.58
$^{40}\text{Ca}+^{42}\text{Ti}$	0.952	9.74	60.34	9.67	60.32	9.48	61.32	9.58	60.98
$^{40}\text{Ca}+^{46}\text{Ti}$	1.048	9.91	59.37	9.86	59.30	9.70	60.07	9.76	59.95
$^{40}\text{Ca}+^{48}\text{Ti}$	1.095	10.00	58.92	9.95	58.84	9.80	59.51	9.84	59.50
$^{40}\text{Ca}+^{50}\text{Ti}$	1.143	10.07	58.49	10.04	58.40	9.90	59.00	9.91	59.09
$^{26}\text{S}+^{52}\text{Ni}$	0.772	9.34	63.79	9.26	63.71	8.94	65.75	9.14	64.80
$^{26}\text{S}+^{56}\text{Ni}$	0.863	9.52	62.71	9.43	62.75	9.13	64.56	9.34	63.55
$^{28}\text{S}+^{52}\text{Ni}$	0.818	9.48	62.95	9.40	62.89	9.14	64.52	9.29	63.81
$^{28}\text{S}+^{56}\text{Ni}$	0.909	9.65	61.96	9.57	61.96	9.32	63.38	9.48	62.68
$^{30}\text{S}+^{52}\text{Ni}$	0.863	9.61	62.18	9.54	62.15	9.31	63.46	9.43	62.94
$^{30}\text{S}+^{56}\text{Ni}$	0.954	9.77	61.27	9.70	61.24	9.50	62.36	9.61	61.91
$^{32}\text{S}+^{58}\text{Ni}$	1.045	9.95	60.23	9.90	60.15	9.74	60.97	9.80	60.08
$^{32}\text{S}+^{64}\text{Ni}$	1.182	10.16	59.12	10.12	59.00	9.97	59.67	9.99	59.75
$^{34}\text{S}+^{58}\text{Ni}$	1.090	10.06	59.65	10.02	59.53	9.88	60.18	9.90	60.20
$^{34}\text{S}+^{64}\text{Ni}$	1.227	10.25	58.61	10.24	58.41	10.11	58.92	10.08	59.21
$^{36}\text{S}+^{58}\text{Ni}$	1.136	10.16	59.11	10.13	58.96	10.01	59.47	10.00	59.65
$^{36}\text{S}+^{64}\text{Ni}$	1.273	10.34	58.12	10.35	57.85	10.24	58.23	10.18	58.72
$^{34}\text{Ar}+^{52}\text{Ni}$	0.870	9.72	69.15	9.65	69.11	9.42	70.54	9.57	69.95
$^{34}\text{Ar}+^{54}\text{Ni}$	0.913	9.80	68.64	9.73	68.60	9.52	69.91	9.65	69.36
$^{34}\text{Ar}+^{56}\text{Ni}$	0.956	9.88	68.15	9.81	68.11	9.61	69.32	9.73	68.34
$^{36}\text{Ar}+^{52}\text{Ni}$	0.913	9.83	68.44	9.76	68.41	9.57	69.62	9.68	69.18
$^{36}\text{Ar}+^{54}\text{Ni}$	0.956	9.91	67.96	9.84	67.91	9.66	69.01	9.76	68.63
$^{40}\text{Ar}+^{58}\text{Ni}$	1.130	10.24	65.92	10.21	65.76	10.08	66.40	10.10	66.51
$^{40}\text{Ar}+^{60}\text{Ni}$	1.174	10.31	65.54	10.29	65.34	10.17	65.92	10.16	66.14
$^{40}\text{Ar}+^{62}\text{Ni}$	1.217	10.37	65.18	10.36	64.93	10.24	65.47	10.22	65.79
$^{40}\text{Ar}+^{64}\text{Ni}$	1.260	10.43	64.82	10.43	64.54	10.32	65.03	10.28	65.46

(a) Based on the AW 95 potential (b) Based on the Bass 80 potential

(c) Based on the Denisov DP potential (d) Based on the Prox. 2010 potential

Table 1. (Continued)

Reaction	N/Z	R_B^a (fm)	V_B^a (MeV)	R_B^b (fm)	V_B^b (MeV)	R_B^c (fm)	V_B^c (MeV)	R_B^d (fm)	V_B^d (MeV)
$^{36}\text{Ca}+^{50}\text{Ni}$	0.791	9.63	77.40	9.55	77.33	9.28	79.27	9.46	78.47
$^{36}\text{Ca}+^{52}\text{Ni}$	0.833	9.72	76.78	9.64	76.73	9.39	78.51	9.56	77.72
$^{36}\text{Ca}+^{54}\text{Ni}$	0.875	9.80	76.18	9.72	76.16	9.49	77.81	9.65	77.04
$^{36}\text{Ca}+^{56}\text{Ni}$	0.916	9.88	75.62	9.80	75.61	9.58	77.15	9.74	76.42
$^{38}\text{Ca}+^{52}\text{Ni}$	0.875	9.83	76.02	9.75	75.98	9.53	77.52	9.68	76.86
$^{38}\text{Ca}+^{54}\text{Ni}$	0.916	9.91	75.46	9.83	75.42	9.63	76.83	9.77	76.23
$^{38}\text{Ca}+^{56}\text{Ni}$	0.958	9.98	74.93	9.91	75.89	9.72	76.20	9.85	75.65
$^{40}\text{Ca}+^{52}\text{Ni}$	0.916	9.93	75.31	9.85	75.28	9.66	76.61	9.79	76.08
$^{40}\text{Ca}+^{54}\text{Ni}$	0.958	10.01	74.78	9.94	74.73	9.75	75.95	9.87	75.50
$^{40}\text{Ca}+^{58}\text{Ni}$	1.042	10.15	73.80	10.10	73.70	9.93	74.74	10.02	74.45
$^{40}\text{Ca}+^{62}\text{Ni}$	1.125	10.29	72.90	10.25	72.76	10.09	73.67	10.15	73.56
$^{40}\text{Ti}+^{48}\text{Ni}$	0.760	9.65	84.93	9.56	84.84	9.28	87.07	9.47	86.20
$^{40}\text{Ti}+^{50}\text{Ni}$	0.8	9.74	84.23	9.65	84.16	9.39	86.20	9.58	85.32
$^{40}\text{Ti}+^{52}\text{Ni}$	0.84	9.82	83.56	9.74	83.51	9.50	85.38	9.68	84.52
$^{40}\text{Ti}+^{56}\text{Ni}$	0.92	9.99	82.32	9.91	82.30	9.69	83.92	9.86	83.14
$^{42}\text{Ti}+^{52}\text{Ni}$	0.88	9.92	82.80	9.84	82.76	9.62	84.41	9.79	83.67
$^{42}\text{Ti}+^{56}\text{Ni}$	0.96	10.08	81.64	10.01	81.58	9.82	82.98	9.98	82.37
$^{48}\text{Ti}+^{58}\text{Ni}$	1.12	10.4	79.30	10.37	79.30	10.23	80.00	10.29	79.95
$^{48}\text{Ti}+^{60}\text{Ni}$	1.16	10.47	78.84	10.45	78.84	10.31	79.43	10.35	79.49
$^{48}\text{Ti}+^{64}\text{Ni}$	1.24	10.60	77.97	10.59	77.97	10.47	78.37	10.47	78.67
$^{46}\text{Ti}+^{64}\text{Ni}$	1.2	10.52	78.49	10.50	78.49	10.37	79.06	10.39	79.19
$^{50}\text{Ti}+^{60}\text{Ni}$	1.2	10.55	78.32	10.54	78.32	10.41	78.77	10.42	78.98
$^{48}\text{Ni}+^{48}\text{Ni}$	0.714	9.72	106.92	9.63	106.69	9.30	109.78	9.59	108.54
$^{50}\text{Ni}+^{50}\text{Ni}$	0.786	9.93	105.10	9.82	104.99	9.54	107.57	9.81	106.32
$^{52}\text{Ni}+^{50}\text{Ni}$	0.821	10.01	104.26	9.91	104.19	9.65	106.56	9.90	105.35
$^{54}\text{Ni}+^{50}\text{Ni}$	0.857	10.09	103.47	10.00	103.42	9.76	105.62	10.00	104.46
$^{54}\text{Ni}+^{54}\text{Ni}$	0.928	10.26	102.00	10.18	101.91	9.97	103.76	10.17	102.81
$^{54}\text{Ni}+^{56}\text{Ni}$	0.964	10.33	101.31	10.26	101.20	10.07	102.92	10.25	102.07
$^{56}\text{Ni}+^{50}\text{Ni}$	0.892	10.18	102.71	10.09	102.69	9.86	104.74	10.08	103.64
$^{56}\text{Ni}+^{52}\text{Ni}$	0.928	10.26	102.00	10.18	101.93	9.96	103.80	10.17	102.83
$^{56}\text{Ni}+^{54}\text{Ni}$	0.964	10.33	101.31	10.26	101.20	10.07	102.92	10.25	102.07
$^{58}\text{Ni}+^{58}\text{Ni}$	1.071	10.55	99.41	10.51	99.18	10.34	100.55	10.47	100.08
$^{58}\text{Ni}+^{64}\text{Ni}$	1.178	10.75	97.71	10.74	97.36	10.59	98.52	10.66	98.42
$^{64}\text{Ni}+^{64}\text{Ni}$	1.286	10.94	96.17	10.96	95.61	10.83	96.57	10.84	96.92

(a) Based on the AW 95 potential (b) Based on the Bass 80 potential

(c) Based on the Denisov DP potential (d) Based on the Prox. 2010 potential

Table 2. The calculated values of constant coefficients α_i and α'_i which are extracted for fitting to regular linear-behavior of R_B , V_C , V_N and V_B , Eqs. (16,17) and (20,21), as a function of increasing neutron in both ranges $N/Z \leq 1$ and $N/Z \geq 1$.

Proximity-model	α_1	α_2	α_3	α_4	α'_1	α'_2	α'_3	α'_4
AW 95	21.24	-20.88	-23.13	51.29	16.18	-14.31	-16.06	31.45
Bass 80	21.86	-20.43	-17.14	57.68	18.26	-15.33	-23.54	41.82
Denisov DP	27.70	-27.52	-32.13	68.33	20.78	-18.11	-19.98	38.56
Prox. 2010	25.00	-24.30	-27.02	55.00	16.57	-13.63	-14.90	28.80

Table 3. The calculated values of constant coefficients β_i and β'_i which are extracted for fitting to regular non-linear-behavior of R_B , V_C , V_N and V_B , Eqs. (18) and (23), as a function of increasing neutron in $0.5 \leq N/Z \leq 1.6$ range.

Proximity-model	β_1	β_2	β_3	β_4	β'_1	β'_2	β'_3	β'_4
AW 95	-11.25	19.38	11.03	-17.55	21.75	-18.08	-28.69	39.86
Bass 80	-1.94	20.37	4.55	-17.74	6.52	-20.26	-14.90	49.67
Denisov DP	-10.87	24.84	17.11	-22.84	20.07	-25.65	-35.30	52.69
Prox. 2010	-15.84	20.35	23.01	-19.31	26.59	-21.30	-41.85	41.88

Table 4. The calculated values of constant coefficients γ which are extracted for fitting to regular linear-behavior of σ_{fus} , Eq. (27), as a function of $N/Z - 1$.

Proximity-model	γ (for $E_{c.m.} = 1.125V_B^0$)	γ (for $E_{c.m.} = 1.375V_B^0$)
AW 95	172.21	80.94
Bass 80	178.82	87.60
Denisov DP	22.31	108.35
Prox. 2010	185.03	95.28

Figure 1:

Figure 2:

Figure 3:

Figure 4:

Figure 5:

Figure 6:

Figure 7: